Executing and Verifying CCS in Maude[*]

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Technical Report TR-SIP-99-00
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February 2000

[*]Research supported by CICYT project *Desarrollo Formal de Sistemas Distribuidos* (TIC97–0669–C03–01).

Abstract

We explore the features of rewriting logic and its language Maude as a logical and semantic framework for representing both the semantics of CCS, and a modal logic for describing local capabilities of CCS processes. Although a rewriting logic representation of the CCS semantics was given in [14], it cannot be directly executed in the current default interpreter of Maude. Moreover, it cannot be used to answer questions such as which are the successors of a process after performing an action, which is used to define the semantics of Hennessy-Milner modal logic. Basically, the problems are the existence of new variables in the righthand side of the rewrite rules and the nondeterministic application of the semantic rules, inherent to CCS. We show how these problems can be solved by exploiting the reflective properties of rewriting logic, which allow controlling the rewriting process. We also show how the semantics can be extended to traces of actions and to the CCS weak transition relation. This executable specification plus the reflective control of the rewriting process can be used to analyze CCS processes.

Contents

1 Introduction

Rewriting logic [17] was introduced in [15] as a unified model of concurrency in which several well-known models of concurrent systems can be represented in a common framework. This goal was further extended in [14] to the idea of rewriting logic as a *logical and semantic framework*. It was shown that many other logics, widely different in nature, can be represented inside rewriting logic in a natural and direct way. The general way in which such representations are achieved is by:

- Representing formulas or, more generally, proof-theoretic structures such as sequents, as terms in an order-sorted equational data type whose equations express structural axioms natural to the logic in question.
- Representing the rules of deduction of a logic as rewrite rules that transform certain patterns of formulas into other patterns modulo the given structural axioms.

Similar techniques can be used to naturally specify and prototype many languages and systems in rewriting logic. In particular, the similarities between rewriting logic and structural operational semantics [20] were noted in [15] and further explored in [14]. As an illustrative example, the paper [14] completely develops a representation of Milner's CCS [19] in rewriting logic, extending ideas first introduced in [18]. However, this representation of CCS cannot be directly executed in the current default interpreter of Maude [5, 6], a high-level language and high-performance system supporting both equational and rewriting logic computation. Maude should be viewed as a *metalanguage* in which the syntax and semantics of other languages, including formal specification languages, can be formally defined [4, 8].

Basically, the problems with the CCS specification are the existence of new variables in the righthand side of the rewrite rules, and the nondeterministic application of the semantic rules, inherent to CCS. We show how these two problems can be solved by exploiting the reflective capabilities of rewriting logic and of Maude, that allow representing rewriting logic inside itself [9, 10], and in particular controlling the rewriting process. Moreover, we show how the semantics can be extended to traces of actions and to the CCS weak transition relation. This executable specification plus the reflective control of the rewriting process can be used to analyze CCS processes and to answer different questions such as which are the successors of a process after performing an action. In summary, we have managed to make the representation of CCS *executable* using reflective techniques in such a way that it can be used to define in Maude the semantics of Hennessy-Milner modal logic [12].

In the rest of this introduction we review the representation of the CCS semantics in Maude, in order to see in detail how the two problems we mentioned above arise. At the same time, we introduce Maude syntax. We use the Full Maude extension [11] to take advantange of parameterization mechanism. To begin with, we show the representation of the CCS syntax in two *functional modules*, that is, equational theories used to specify algebraic data types, defining actions and processes. We declare `sort(s)`, `subsort(s)`, and operators `op(s)`, which have user-definable syntax.[1] The operators precedence is set by

[1]Due to parsing restrictions, some characters (`[ ] { } ( ) ,`) have to be preceded by character `‘` when declaring them in Full Maude.

means of the attribute `prec`. Equations are declared with the keywords `eq` or `ceq` (for conditional ones). The imported module `QID` is a useful built-in module providing quoted identifiers that in this case represent CCS labels and process identifiers.

```
(fmod ACTION is
  protecting QID .
  sorts Label Act .
  subsorts Qid < Label < Act .

  op tau : -> Act .    *** silent action

  op ~_ : Label -> Label .
  var N : Label .
  eq ~ ~ N = N .
endfm)

(fmod PROCESS is
  protecting ACTION .
  sorts ProcessId Process .
  subsort Qid < ProcessId < Process .

  op 0 : -> Process .                              *** inaction
  op _._ : Act Process -> Process [prec 25] .      *** prefix
  op _+_ : Process Process -> Process [prec 35] .  *** summation
  op _|_ : Process Process -> Process [prec 30] .  *** composition
  op _`[_/_`] : Process Label Label -> Process [prec 20] .
                        *** relabelling: [b/a] relabels "a" to "b"
  op _\_ : Process Label -> Process [prec 20] .    *** restriction
endfm)
```

Full CCS is represented, including (possibly recursive) process definitions by means of *contexts*. A context is well-formed if a process identifier is defined at most once. We use a conditional membership axiom (`cmb`) to establish which terms of sort `BadContext` are well-formed contexts (of sort `Context`). The evaluation of `def(X,C)` returns the process associated to process identifier `X` if it exists; otherwise, it returns the bad process `not-defined`.

```
(fmod CCS-CONTEXT is
  protecting PROCESS .
  sorts BadProcess Context BadContext .
  subsort Process < BadProcess .
  subsort Context < BadContext .

  op _=def_ : ProcessId Process -> Context [prec 40] .
  op nil : -> Context .
  op _&_ : BadContext BadContext -> BadContext
            [assoc comm id: nil prec 42] .
  op _definedIn_ : ProcessId Context -> Bool .
  op def : ProcessId Context -> BadProcess .
  op not-defined : -> BadProcess .
  op context : -> Context .

  vars X X' : ProcessId .
  var P : Process .
  var C : Context .

  cmb (X =def P) & C : Context if not(X definedIn C) .

  eq X definedIn nil = false .
  eq X definedIn ((X' =def P) & C) = (X == X') or (X definedIn C) .

  eq def(X, nil) = not-defined .
  eq def(X, ((X =def P) & C)) = P .
  ceq def(X, ((X' =def P) & C)) = def(X, C) if X =/= X' .
endfm)
```

The general idea, as stated in [14], for implementing in rewriting logic the operational semantics of CCS, is to translate each semantic rule either into a rewrite rule where the premises are rewritten to the conclusion, or into a rewrite rule where the conclusion is rewritten to the premises. In [14] the first approach was followed. In this paper we adopt the second one, because we want to be able to prove in a bottom-up way that a given transition is valid in CCS. There is another possibility, that is not considered in this paper, where the transition $P \xrightarrow{a} P'$ is interpreted as a rewrite, so that the operational semantic rules become conditional rewrite rules where the premises form the condition [14].

The CCS transition judgement $P \xrightarrow{a} P'$ is represented in Maude by the term P `--` a `->` P' of sort `Judgement`, built by means of the operator

```
  sort Judgement .
  op _--_->_ : Process Act Process -> Judgement [prec 50] .
```

In general, a semantic rule has a conclusion and a set of premises, each one represented by a judgement. So we need a sort to represent sets of judgements:

```
  sort JudgementSet .
  subsort Judgement < JudgementSet .

  op emptyJS : -> JudgementSet .
  op __ : JudgementSet JudgementSet -> JudgementSet
          [assoc comm id: emptyJS prec 60] .
```

The union constructor is written with empty syntax (`__`), and declared associative (`assoc`), commutative (`comm`), and with the empty set as identity element (`id: emptyJS`). Matching and rewriting take place *modulo* such properties. Idempotency is specified by means of an explicit equation.

```
  var J : Judgement .
  eq J J = J .
```

A semantic rule is implemented as a rewrite rule where the singleton set consisting of the judgement representing the conclusion is rewritten to the set consisting of the judgements representing the premises. Rewrite rules (introduced with the keywords `rl` or `crl`) are declared in *system modules*, which are rewrite theories specifying concurrent systems. Hence, system modules define the *dynamic* aspects of systems, whereas functional modules define *static* data types.

For example, for the restriction operator of CCS, we have the semantic rule,

$$\frac{P \xrightarrow{a} P'}{P \setminus l \xrightarrow{a} P' \setminus l}\ [a \neq l \wedge a \neq \bar{l}]$$

which is translated to the following rewrite rule where, using the fact that text beginning with `---` is a comment, the rule is displayed in such a way as to emphasize the correspondence with the usual presentation in textbooks:

```
crl [res] :  P \ L -- A -> P' \ L
        => ------------------------
                 P -- A -> P'
        if (A =/= L) and (A =/= ~ L) .
```

As another example, the axiom schema

$$\frac{}{a.P \xrightarrow{a} P}$$

defining the behaviour of the prefix operator gives rise to the rewrite rule

```
rl [pref] :   A . P -- A -> P
        => ---------------------
                  emptyJS .
```

Thus, a transition $P \xrightarrow{a} P'$ is possible in CCS if and only if the judgement representing it can be rewritten to the empty set of judgements by rewrite rules of the form described above that define the operational semantics of CCS in a backwards search fashion.

However, we have found two problems while working with this approach in the current version of the Maude system. The first one is that sometimes new variables appear in the premises which are not present in the conclusion. For example, in one of the semantic rules for the parallel operator we have

```
rl [par] :     P | Q -- tau -> P' | Q'
        => --------------------------------
               P -- L -> P'   Q -- ~ L -> Q' .
```

where `L` is a new variable in the righthand side of the rewrite rule. Rules of this kind cannot be directly used by the current Maude default interpreter: they can only be used at the metalevel using a strategy to instantiate the extra variables.

Another problem is that sometimes several rules can be applied to rewrite a judgement. For example, for the summation operator we have, because of its intrinsic nondeterminism,

```
rl [sum] :   P + Q -- A -> P'
        => -------------------
               P -- A -> P' .

rl [sum] :   P + Q -- A -> Q'
        => -------------------
               Q -- A -> Q' .
```

Only one rule is enough by declaring the operator `_+_` to be commutative. However, in the commutative case, nondeterminism appears because of possible multiple matches (modulo commutativity) against the pattern `P + Q`.

In general, not all of these possibilities lead to an empty set of judgements. So we have to deal with the whole tree of possible rewritings of a judgement, searching if one of the branches leads to `emptyJS`.

In Section 2, we show how these problems can be solved in the current version of the Maude system by using reflection, obtaining an executable semantics, where we can prove if a transition $P \xrightarrow{a} P'$ is possible.

In Section 3, we extend this representation in order to be able to answer a different kind of questions, such as if process P can perform action a (and we do not care about the process it becomes), or which are the *successors* of a process P after performing actions in a given set As, that is,

$$succ(P, As) = \{P' \mid P \xrightarrow{a} P' \wedge a \in As\}.$$

In Sections 4 and 5 we extend the CCS semantics to sequences of actions (or *traces*) and to the weak transition semantics, which does not observe τ transitions.

In Section 6 we show how we can define in Maude the semantics of the Hennessy-Milner modal logic for describing local capabilities of CCS processes.

2 Executable CCS semantics in Maude

In this section we show how the problem of new variables in the righthand side of a rewrite rule is solved by using the concept of *explicit metavariables* presented in [22], and how nondeterministic rewriting is controlled by means of a *search strategy* [2, 7].

2.1 Definition of the executable semantics

New variables in the righthand side of a rewrite rule represent "unknown" values when we are rewriting; by using metavariables we make explicit this lack of knowledge. The semantics with explicit metavariables has to bind them to concrete values when these values become known.

For the time being, metavariables are only needed as actions in the judgements, so we declare a new sort for metavariables as actions:

```
sort MetaVarAct .
op ?‘(_‘)A : Qid -> MetaVarAct .
var NEW1 : Qid .
```

We also introduce a new sort `Act?` of "possible actions,"

```
sort Act? .
subsorts Act MetaVarAct < Act? .
var ?A : MetaVarAct .
var A? : Act? .
```

and modify the operator for building judgements in order to deal with this new sort of actions:

```
op _--_->_ : Process Act? Process -> Judgement [prec 50] .
```

As mentioned above, a metavariable will be bound when its concrete value becomes known, so we need a new judgement stating that a metavariable is bound to a concrete value

```
op `[_:=_`] : MetaVarAct Act -> Judgement .
```

and a way to propagate this binding to the rest of judgements where the bound metavariable may be present. Since this propagation has to reach all the judgements in the current state of the inference process, we introduce an operation to enclose the set of judgements, and a rule to propagate a binding

```
op `{`{_`}`} : JudgementSet -> Configuration .

var JS : JudgementSet .
rl [bind] : {{ [?A := A] JS }} => {{ <act ?A := A > JS }} .
```

where we use the following auxiliary functions in order to perform the corresponding substitutions

```
op <act_:=_>_ : MetaVarAct Act Act? -> Act? .
op <act_:=_>_ : MetaVarAct Act Judgement -> Judgement .
op <act_:=_>_ : MetaVarAct Act JudgementSet -> JudgementSet .
```

Now we are able to redefine the rewrite rules implementing the CCS semantics, taking care of metavariables. For the prefix operator we maintain the previous axiom

```
rl [pref] :  A . P -- A -> P
        => --------------------
                 emptyJS .
```

and add a new rule for the case when a metavariable appears in the judgement

```
rl [pref] :  A . P -- ?A -> P
        => --------------------
                [?A := A] .
```

Note how the metavariable `?A` present in the lefthand side judgement is bound to the concrete action `A` taken from the process `A . P`. This binding will be propagated to any other judgement in the set of judgements containing `A . P -- ?A -> P`.

For the summation operator, we generalize the rules allowing a more general variable `A?` of sort `Act?`, since the behaviour is the same independently of whether a metavariable or an action appears in the judgement:

```
rl [sum] :  P + Q -- A? -> P'
        => -------------------
               P -- A? -> P' .

rl [sum] :  P + Q -- A? -> Q'
        => -------------------
               Q -- A? -> Q' .
```

Nondeterminism is again present; we will deal with it in Section 2.3.

For the parallel operator, there are two rules for the cases when one of the composed processes performs an action on its own,

```
rl [par] :  P | Q -- A? -> P' | Q
        => ----------------------
                P -- A? -> P' .

rl [par] :  P | Q -- A? -> P | Q'
        => -----------------------
                Q -- A? -> Q' .
```

and two additional rules dealing with the case when communication happens between both processes,

```
rl [par] :           P | Q -- tau -> P' | Q'
        => ---------------------------------------------
            P -- ?(NEW1)A -> P'   Q -- ~ ?(NEW1)A -> Q' .

rl [par] :                P | Q -- ?A -> P' | Q'
        => -----------------------------------------------------------
            P -- ?(NEW1)A -> P'   Q -- ~ ?(NEW1)A -> Q'  [?A := tau] .
```

where we have overloaded the ~ operator

```
op ~_ : Act? -> Act? .
```

Note how the term `?(NEW1)A` is used to represent a *new metavariable.* Rewriting has to be controlled by a *strategy* that instantiates the variable `NEW1` with a new (quoted) identifier each time one of the above rules is applied, in order to build *new* metavariables. The strategy presented in Section 2.3 does this besides implementing the search in the tree of possible rewritings.

There are two rules dealing with the restriction operator of CCS: one for the case when an action occurs in the lefthand side judgement,

```
crl [res] :  P \ L -- A -> P' \ L
        => -----------------------
                P -- A -> P'
        if (A =/= L) and (A =/= ~ L) .
```

and another one for the case when a metavariable occurs in the lefthand side judgement,

```
rl [res] :          P \ L -- ?A -> P' \ L
       => -----------------------------------------
           P -- ?A -> P'  [?A =/= L]  [?A =/= ~ L] .
```

In the latter case, we cannot use a conditional rewrite rule as in the former case, because the corresponding condition `(?A =/= L) and (?A =/= ~ L)` cannot be checked until we know the concrete value of the metavariable `?A`. Hence, we have to add a new kind of judgements

```
  op `[_=/=_`] : Act? Act? -> Judgement .
```

used to state constraints related with metavariables (which will be substituted by actions). This constraint is eliminated when it is fulfilled,

```
  crl [dist] : [A =/= A'] => emptyJS  if A =/= A' .
```

where (normal) actions are used.

For the relabelling operator of CCS we have similar rewrite rules:

```
  rl [rel] :  P[M / L] -- M -> P'[M / L]
          => -----------------------------
                      P -- L -> P' .

  rl [rel] :  P[M / L] -- ~ M -> P'[M / L]
          => -------------------------------
                     P -- ~ L -> P' .

  crl [rel] :  P[M / L] -- A -> P'[M / L]
           => -----------------------------
                      P -- A -> P'
           if (A =/= L) and (A =/= ~ L) .

  rl [rel] :  P[M / L] -- ?A -> P'[M / L]
          => -----------------------------
                 (P -- L -> P') [?A := M] .

  rl [rel] :  P[M / L] -- ?A -> P'[M / L]
          => -------------------------------
               P -- ~ L -> P'  [?A := ~ M] .

  rl [rel] :      P[M / L] -- ?A -> P'[M / L]
          => ------------------------------------------
                P -- ?A -> P' [?A =/= L] [?A =/= ~ L] .
```

Finally, process identifiers only need the generalization of the original rule by means of a more general variable `A?`.

```
  crl [def] :         X -- A? -> P'
           => -------------------------------
                def(X, context) -- A? -> P'    if (X definedIn context) .
```

Using the above rules, we can begin to pose some questions about the capability of a process to perform an action. For example, we can ask if the process `'a . 'b . 0` can perform action `'a` (becoming process `'b . 0`) by rewriting the configuration composed of a judgement representing that transition:

```
Maude> (rew {{ 'a . 'b . 0 -- 'a -> 'b . 0 }} .)
result Configuration : {{ emptyJS }}
```

Since a configuration consisting of the empty set of judgements is reached, we can conclude that the transition is possible.

However, if we ask if the process `'a . 0 + 'b . 0` can perform action `'b` becoming process `0`, we get as result

```
Maude> (rew {{ 'a . 0 + 'b . 0 -- 'b -> 0 }} .)
result Configuration : {{ 'a . 0 -- 'b -> 0 }}
```

representing that the given transition is not possible, which is not the case. The problem is that the configuration `{{ 'a . 0 + 'b . 0 -- 'b -> 0 }}` can be rewritten in two different ways, and only one of them leads to a configuration consisting of the empty set of judgements, as shown in the following tree:

$$\begin{tikzcd}
 & \texttt{\{\{ 'a . 0 + 'b . 0 -- 'b -> 0 \}\}} \arrow[dl] \arrow[dr] & \\
\texttt{\{\{ 'a . 0 -- 'b -> 0 \}\}} & & \texttt{\{\{ 'b . 0 -- 'b -> 0 \}\}} \arrow[d] \\
 & & \texttt{\{\{ emptyJS \}\}}
\end{tikzcd}$$

Therefore, we need a strategy to search the tree of all possible rewrites.

2.2 Maude's Metalevel

Rewriting logic is reflective [9, 10, 3], that is, there is a finitely presented rewrite theory $\mathcal{U}$ that is *universal* in the sense that we can represent any finitely presented rewrite theory $\mathcal{R}$ (including $\mathcal{U}$ itself) and any terms t, t' in $\mathcal{R}$ as terms $\overline{\mathcal{R}}$ and $\overline{t}, \overline{t'}$ in $\mathcal{U}$, and we then have the following equivalence

$$\mathcal{R} \vdash t \longrightarrow t' \;\;\Leftrightarrow\;\; \mathcal{U} \vdash \langle \overline{\mathcal{R}}, \overline{t} \rangle \longrightarrow \langle \overline{\mathcal{R}}, \overline{t'} \rangle.$$

In Maude, key functionality of the universal theory $\mathcal{U}$ has been efficiently implemented in the functional module `META-LEVEL`, where

- Maude terms are reified as elements of a data type `Term` of terms;
- Maude modules are reified as terms in a data type `Module` of modules;
- the processes of reducing a term to normal form in a functional module and of finding whether such a normal form has a given sort are reified by a metalevel function `meta-reduce`;
- the process of applying a rule of a system module to a subject term is reified by a metalevel function `meta-apply`;
- the process of rewriting a term in a system module using Maude's default interpreter is reified by a metalevel function `meta-rewrite`; and
- parsing and pretty printing of a term in a signature, as well as key sort operations such as comparing sorts in the subsort ordering of a signature, are also reified by corresponding metalevel functions.

2.3 Searching in the tree of rewritings

In this section we show how the reflective properties of Maude [10] can be used to control the rewriting of a term, and the search in the tree of possible rewritings of a term. The depth-first strategy is based on the work in [1, 2, 7], modified to deal with the substitution of metavariables explained in the previous section.

The module implementing the search strategy is parameterized with respect to a constant equal to the metarepresentation of the Maude module which we want to work with. Hence, we define a parameter theory with a constant `MOD` representing the module, and a constant `labels` representing the list of labels of rewrite rules to be applied:

```
(fth AMODULE is
  including META-LEVEL .
  op MOD : -> Module .
  op labels : -> QidList .
endfth)
```

The module containing the strategy, extending `META-LEVEL`, is then the parameterized module `SEARCH[M :: AMODULE]`.

Since we are defining a strategy to search a tree of possible rewritings, we need a notion of search goal. For the strategy to be general enough, we assume that the metarepresented module `MOD` has an operation `ok` (defined at the object level, see Section 2.4), which returns a value of sort `Answer` such that

- `ok(T) = solution` means that the term `T` is one of the terms we are looking for, that is, `T` denotes a solution;
- `ok(T) = no-solution` means that the term `T` is not a solution and no solution can be found below `T` in the search tree;
- `ok(T) = maybe-sol` means that `T` is not a solution, but we do not know if there are solutions below it.

The strategy controls the possible rewritings of a term by means of the metalevel function `meta-apply`. The evaluation of `meta-apply(MOD,T,L,S,N)` applies (discarding the first `N` successful matches) a rule of module `MOD` with label `L`, partially instantiated with substitution `S`, to the term `T` (at the top level). It returns the resulting fully reduced term and the representation of the match used in the reduction, `{ T', S' }`.

In Section 2.1 we saw the necessity of instantiating the new variables in the righthand side of a rewrite rule in order to create new metavariables. We have to provide a substitution in such a way that the rules are always applied without new variables in the righthand side. For simplicity we will assume that a rule has at most three new variables called `NEW1`, `NEW2`, and `NEW3`. (However, the version given in Appendix A is completely general, for any number of new variables.) These variables are then substituted by new identifiers, which are quoted numbers. Hence, we define a new operation `meta-apply'` which receives the greatest number `M` used to substitute variables in `T` and uses three new numbers to create three new (metarepresented) identifiers.

```
var N M : MachineInt .
var L : : Qid .
var T : Term .
var SB : Substitution .

op new-var : MachineInt -> Term .
eq new-var(M) = {conc('' , index(' , M))}'Qid .

op createSubst : MachineInt -> Substitution .
eq createSubst(M) = (('NEW1@Qid <- new-var(M + 1));
                     ('NEW2@Qid <- new-var(M + 2));
                     ('NEW3@Qid <- new-var(M + 3))) .

op extTerm : ResultPair -> Term .
eq extTerm({T, SB}) = T .

op meta-apply' : Term Qid MachineInt MachineInt -> Term .
eq meta-apply'(T, L, N, M) = extTerm(meta-apply(MOD, T, L, createSubst(M), N)) .
```

The operation `meta-apply'` returns *one* of the possible one-step rewritings at the top level of a given term. Our first step is to define an operation `allRew` that returns *all* the possible *one-step sequential* rewritings [15] of a given term `T` by using rewrite rules with labels in the list `labels`. The third argument of `allRew` represents the greatest number `M` used to substitute new variables in `T`. There is a `TermList` sort in module `META-LEVEL`, but it does not have an identity element, that we need to represent the case when no rule can be applied. So we extend it as follows:

```
op ~ : -> TermList .
var TL : TermList .
eq ~, TL = TL .
eq TL, ~ = TL .
```

The operations needed to find all the possible rewritings, and their definitions, are as follows:

```
op allRew : Term QidList MachineInt  -> TermList .
op topRew : Term Qid MachineInt MachineInt -> TermList .
op lowerRew : Term Qid MachineInt -> TermList .
op rewArguments : Qid TermList TermList Qid MachineInt -> TermList .
op rebuild : Qid TermList TermList TermList -> TermList .

var LS : QidList .
vars C S OP : Qid .
vars Before After : TermList .

eq allRew(T, nil, M) = ~ .
eq allRew(T, L LS, M) = topRew(T, L, 0, M), *** rewriting at the top of T
                        lowerRew(T, L, M),  *** rewriting of (proper) subterms
                        allRew(T, LS, M) .  *** rewriting with labels LS
```

The evaluation of `topRew(T, L, N, M)` returns all the possible one-step rewritings at the top level of term `T` applying rule `L`, discarding the first `N` matches, and using numbers from `M+1` to create identifiers for new variables.

```
eq topRew(T, L, N, M) =
    if meta-apply'(T, L, N, M) == error* then ~
    else (meta-apply'(T, L, N, M) , topRew(T, L, N + 1, M)) fi .
```

The evaluation of `lowerRew(T,L,M)` returns all the possible one-step rewritings of the subterms of term `T` applying rule `L`, and using numbers from `M+1` to create identifiers for new variables. If `T` is a constant (term without arguments), the empty list of terms is returned; otherwise, the operation `rewArguments` is used, which is defined recursively on its third argument, which represents the arguments of `T` not rewritten yet.

```
eq lowerRew({C}S, L, M) = ~ .
eq lowerRew(OP[TL], L, M) = rewArguments(OP, ~, TL, L, M) .

eq rewArguments(OP, Before, T, L, M) =
    rebuild(OP, Before, allRew(T, L, M), ~) .
eq rewArguments(OP, Before, (T, After), L, M) =
    rebuild(OP, Before, allRew(T, L, M), After) ,
    rewArguments(OP, (Before, T), After, L, M) .
```

The evaluation of `rebuild(OP, Before, TL, After)` returns all the terms of the form `OP[Before, T, After]` where `T` is a term in the term list `TL`. These built terms are metareduced before being returned.

```
eq rebuild(OP, Before, ~, After) = ~ .
eq rebuild(OP, Before, T, After) = meta-reduce(MOD, OP[Before, T, After]) .
eq rebuild(OP, Before, (T, TL), After) =
    meta-reduce(MOD, OP[Before, T, After]), rebuild(OP, Before, TL, After) .
```

For example, we can apply the operation `allRew` to the metarepresentation of the term `{{ 'a . 0 + 'b . 0 -- 'b -> 0 }}` to calculate all its rewritings, and we get the metarepresentation of the terms `{{ 'a . 0 -- 'b -> 0 }}` and `{{ 'b . 0 -- 'b -> 0 }}`.

`Maude> (red allRew(` $\overline{\texttt{\{\{ 'a . 0 + 'b . 0 -- 'b -> 0 \}\}}}$`, labels, 0 ) . )`

`result TermList :` $\overline{\texttt{\{\{ 'a . 0 -- 'b -> 0 \}\}}}$, $\overline{\texttt{\{\{ 'b . 0 -- 'b -> 0 \}\}}}$

Now we can define a strategy to search in the (conceptual) tree of all possible rewritings of a term `T` for a term that satisfies the `ok` predicate. Each node of the search tree is a pair, whose first component is a term and whose second component is a number representing the greatest number used as identifier for new variables in the process of rewriting the term. The tree nodes that have been generated but not yet been checked are maintained in a sequence.

```
sorts Pair PairSeq .
subsort Pair < PairSeq .
op <_`,_> : Term MachineInt -> Pair .
op nil : -> PairSeq .
op _|_ : PairSeq PairSeq -> PairSeq [assoc id: nil] .
var PS : PairSeq .
```

We need an operation to build these pairs from the list of terms produced by `allRew`:

```
  op buildPairs : TermList MachineInt -> PairSeq .

  eq buildPairs(~, N) = nil .
  eq buildPairs(T, N) = < T , N > .
  eq buildPairs((T, TL), N) = < T , N > | buildPairs(TL, N) .
```

The operation `rewDepth` starts the search by calling the operation `rewDepth'` with the root of the search tree. `rewDepth'` returns the first solution found in a depth-first way. If there is no solution, the `error*` term is returned.

```
  op rewDepth : Term -> Term .
  op rewDepth' : PairSeq -> Term .

  eq rewDepth(T) = rewDepth'(< meta-reduce(MOD, T), 0 >) .

  eq rewDepth'(nil) = error* .
  eq rewDepth'(< T , N > | PS) =
      if meta-reduce(MOD, 'ok[T]) == {'solution}'Answer then T
      else (if meta-reduce(MOD, 'ok[T]) == {'no-solution}'Answer then
                 rewDepth'(PS)
            else rewDepth'(buildPairs(allRew(T, labels, N), (N + 3)) | PS )
            fi)
      fi .
```

2.4 Some examples

Now we can test the CCS semantics with some examples using different judgements. First, we define a module `CCS-OK` extending the CCS syntax and the CCS semantic rules by defining some process constants to be used in the examples, and the predicate `ok` that states when a configuration is a solution. In this case a configuration denotes a solution when it is the empty set of judgements, representing that the set of judgements at the beginning is provable by means of the semantic rules.

```
(mod CCS-OK is
  protecting CCS-SEMANTICS .

  ops p1 p2 p3 : -> Process .

  eq p1 = ('a . 0) + ('b . 0 | ('c . 0 + 'd . 0)) .
  eq p2 = ('a . 'b . 0 | (~ 'c . 0) ['a / 'c]) \ 'a .

  eq context = ('Proc =def 'a . tau . 'Proc) .

  eq p3 = ('Proc | ~ 'a . 'b . 0) \ 'a .

  sort Answer .
  ops solution  no-solution  maybe-sol : -> Answer .

  op ok : Configuration -> Answer .
  var JS : JudgementSet .
  eq ok({{ emptyJS }}) = solution .
  ceq ok({{ JS }}) = maybe-sol if JS =/= emptyJS .
endm)
```

In order to instantiate the generic module `SEARCH`, we need the metarepresentation of module `CCS-OK`. We use the Full Maude `up` function [5, 11] to obtain the metarepresentation of a module or a term.

```
(mod META-CCS is
  including META-LEVEL .
  op METACCS : -> Module .
  eq METACCS = up(CCS-OK) .
endm)
```

We declare a view and instantiate the generic module `SEARCH` with it.

```
(view ModuleCSS from AMODULE to META-CCS is
  op MOD to METACCS .
  op labels to ('bind 'pref 'sum 'par 'res 'dist 'rel 'def) .
endv)

(mod SEARCH-CCS is
  protecting SEARCH[ModuleCCS] .
endm)
```

Now we can test the examples. First we can prove that process `p1` can perform action `'c` becoming `'b . 0 | 0`:

```
Maude> (red rewDepth({{ p1 -- 'c -> 'b . 0 | 0 }})) .)
result Term : {{ emptyJS }}
```

We can also prove that process `p2` cannot perform action `'a` (but see later)

```
Maude> (red rewDepth({{ p2 -- 'a -> ('b . 0 | (~ 'c . 0) ['a / 'c]) \ 'a }})) .)
result Term : error*
```

Process `p2` can perform action `tau` becoming process `('b . 0 | 0 ['a / 'c]) \ 'a`:

```
Maude> (red rewDepth({{ p2 -- tau -> ('b . 0 | 0 ['a / 'c]) \ 'a }})) .)
result Term : {{ emptyJS }}
```

In the same way, we can prove that process `p3` can perform action `tau`

```
Maude> (red rewDepth({{ p3 -- tau -> (tau . 'Proc | 'b . 0) \ 'a }})) .)
result Term : {{ emptyJS }}
```

In all these examples, we have had to provide the resulting process. In the positive proofs there is no problem (besides the cumbersome task of explicitly writing the resulting process), but in the negative proof, that is, that `p2` cannot perform action `'a`, the given proof is not completely correct: We have proved that process `p2` cannot perform action `'a` *becoming* `('b . 0 | (~ 'c . 0) ['a / 'c]) \ 'a`, but we have not proved that there is no way in which `p2` can execute action `'a`. We will see in Section 3.2 how this can be proved.

3 How to obtain new kinds of results

We are now interested in answering questions such as: Can process P perform action a (without caring about the process it becomes)? That is, we want to know if $P \xrightarrow{a} P'$ is possible, but P' is *unknown*. This is the same problem we found when new variables appear in the premises of a semantic rule. The solution, as we did with actions, is to define metavariables as processes.

3.1 Including metavariables as processes

As we did with actions, we declare a new sort of metavariables as processes,

```
  sort MetaVarProc .
  op ?`(_`)P : Qid -> MetaVarProc .
```

add a new sort of possible processes,

```
  sort Process? .
  subsorts Process MetaVarProc < Process? .
  var ?P : MetaVarProc .
  var P? : Process? .
```

and modify the operation to build the basic transition judgements

```
  op _--_->_ : Process? Act? Process? -> Judgement [prec 50] .
```

For the prefix operator we have to add two new rules:

```
  rl [pref] :  A . P -- A -> ?P
          => --------------------
                  [?P := P] .

  rl [pref] :   A . P -- ?A -> ?P
          => -----------------------
               [?A := A] [?P := P] .
```

where, as in the case of metavariables as actions, we have to define a new kind of judgements that bind metavariables with processes, a rule to propagate these bindings, and operations that perform the substitution:

```
  op `[_:=_`] : MetaVarProc Process? -> Judgement .

  rl [bind] : {{ [?P := P] JS }} => {{ <proc ?P := P > JS }} .

  op <proc_:=_>_ : MetaVarProc Process Process? -> Process?  .
  op <proc_:=_>_ : MetaVarProc Process Judgement -> Judgement .
  op <proc_:=_>_ : MetaVarProc Process JudgementSet -> JudgementSet .
```

For the rest of CCS operators, new rules have to be added to deal with metavariables in the second process of the transition judgement (see the complete set of rules in Appendix A).

3.2 More examples

Now, we can prove that process `p1` can perform action `'c`, by rewriting the judgement `p1 -- 'c -> ?('any)P`, where the metavariable `?('any)P` means that we do not care about the resulting process.

```
Maude> (red rewDepth({{ p1 -- 'c -> ?('any)P }}) .)
result Term : {{ emptyJS }}
```

We can also prove that process `p2` *cannot* perform action `'a`

```
Maude> (red rewDepth({{ p2 -- 'a -> ?('any)P }}) .)
result Term : error*
```

3.3 Successors of a process

Another interesting question is which are the *successors* of a process P after performing actions in a given set As, that is,

$$succ(P, As) = \{P' \mid P \xrightarrow{a} P' \wedge a \in As\}.$$

Since we can use metavariables as processes, we can rewrite `P -- A -> ?('proc)P` instantiating the variable `A` with actions in the given set. Those rewritings will bind the metavariable `?('proc)P` with the successors of `P`, but we find two problems. The first one is that we lose the bindings between metavariables and processes when they are substituted by applying the rewrite rule `bind`. To solve this, we have to modify the operator to build configurations, by keeping a set of bindings already produced in addition to the set of judgements to be reduced

```
  op `{`{_|_`}`} : JudgementSet JudgementSet -> Configuration  .
```

The bindings will be saved in the second argument by the `bind` rule:

```
  rl [bind] : {{ [?P := P] JS | JS' }} =>
              {{ (<proc ?P := P > JS) | [?P := P] JS' }} .
```

We have to change also the function `ok`

```
  vars JS JS' : JudgementSet .

  eq ok({{ emptyJS | JS' }}) = solution .
  ceq ok({{ JS | JS'}}) = maybe-sol if JS =/= emptyJS .
```

Another problem is that `rewDepth` only returns one solution, but we can modify it in order to get all the solutions, that is, in order to explore (in a depth-first way) the whole tree of rewritings finding all the nodes that satisfy the function `ok`. The operation `allSol`, added to the module `SEARCH`, returns a set with the terms representing all the solutions.

```
  sort TermSet .
  subsort Term < TermSet .

  op `{`} : -> TermSet .
  op _U_ : TermSet TermSet -> TermSet [assoc comm id: {}] .
  ceq T U T' = T if meta-reduce(MOD, '_==_[T, T']) == {'true}'Bool .

  op allSol : Term -> TermSet .
  eq allSol(T) = allSolDepth(< meta-reduce(MOD,T) , 0 >) .

  op allSolDepth : PairSeq -> TermSet .

  eq allSolDepth(nil) = {} .
  eq allSolDepth(< T , N > | PS) =
    if meta-reduce(MOD, 'ok[T]) == {'solution}'Answer then
              (T U allSolDepth(PS))
    else (if meta-reduce(MOD, 'ok[T]) == {'no-solution}'Answer then
            allSolDepth(PS)
         else allSolDepth(buildPairs(allRew(T, labels, N), (N + 3)) | PS)
         fi)
    fi .
```

Now we can define (in an extension of module `SEARCH-CCS`, see module `CCS-SUCC` in Appendix A) an operation `succ` which, given the metarepresentation of a process and a set of metarepresentations of actions, returns the set of metarepresentations of the process successors.

```
  op succ : Term TermSet -> TermSet .
  eq succ(T, {}) = {} .
  eq succ(T, A U AS) = filter(allSol(
              '`{`{_|_`}`} [ '_--_->_ [ T, A, '?`(_`)P [ {''proc}'Qid ]] ,
                { 'emptyJS } 'JudgementSet ]), '?`(_`)P [ {''proc}'Qid ])
       U succ(T,AS) .
```

where `filter` (see Appendix A) is used to remove all the bindings involving metavariables different from `?('proc)P`.

We can illustrate how these functions work with the processes defined in module `CCS-OK` in Section 2.4. We can calculate the successors of process `p1` after performing action `'a` or action `'b`:

Maude> (red succ($\overline{\texttt{p1}}$, $\overline{\texttt{'a}}$ U $\overline{\texttt{'b}}$) .)

result TermSet : $\overline{\texttt{0 | ('c . 0 + 'd . 0)}}$ U $\overline{\texttt{0}}$

4 Extending the semantics to traces

In the CCS semantics we have used until now, only judgements of the form $P \xrightarrow{a} P'$ exist. We can extend it to sequences of actions or *traces*, that is, to judgements of the form $P \xrightarrow{a_1 a_2 \ldots a_n} P'$. The semantic rules defining these new transitions are

$$\frac{}{P \xrightarrow{\varepsilon} P} \qquad\qquad \frac{P \xrightarrow{a_1} P' \quad P' \xrightarrow{a_2 \ldots a_n} P''}{P \xrightarrow{a_1 a_2 \ldots a_n} P''}$$

where ε denotes the empty trace.

We can extend our framework by defining a new sort for traces

```
sort Trace .
subsort Act < Trace .
op nil : -> Trace .
op __ : Trace Trace -> Trace [assoc id: nil] .
```

and a new kind of judgements

```
op _-_->_ : Process Trace Process -> Judgement [prec 50] .
```

Then, we can translate the above semantic rules to corresponding rewrite rules in an obvious way:

```
var Tr : Trace .

rl [nil] :     P - nil -> P
        => -------------------
                emptyJS .

rl [seq] :             P - A Tr -> P'
        => ------------------------------------------
               P -- A -> ?(NEW1)P   ?(NEW1)P - Tr -> P' .
```

As we have done with judgements dealing with actions, we can also extend judgements dealing with traces in such a way that metavariables can be used as processes. We only have to modify the operation for building this kind of judgements

```
op _-_->_ : Process? Trace Process? -> Judgement [prec 50] .
```

and add new rules for the case when a metavariable appears as the second process

```
rl [nil] :  P - nil -> ?P
        => ---------------
              [?P := P] .

rl [seq] :            P - A Tr -> ?P
        => -------------------------------------------
               P -- A -> ?(NEW1)P   ?(NEW1)P - Tr -> ?P .
```

5 Extension to weak transition semantics

Another important transition relation defined in CCS, $P \overset{a}{\Longrightarrow} P'$, does not observe τ transitions. It is defined as

$$\frac{P\ (\xrightarrow{\tau})^*\ Q \quad Q \xrightarrow{a} Q' \quad Q'\ (\xrightarrow{\tau})^*\ P'}{P \overset{a}{\Longrightarrow} P'}$$

where $(\xrightarrow{\tau})^*$ denotes the reflexive, transitive closure of $\xrightarrow{\tau}$, and it is defined in the following way

$$\frac{}{P\ (\xrightarrow{a})^*\ P} \qquad\qquad \frac{P \xrightarrow{a} P' \quad P'\ (\xrightarrow{a})^*\ P''}{P\ (\xrightarrow{a})^*\ P''}$$

The weak transition semantics is extended to traces as follows:

$$\frac{P\ (\xrightarrow{\tau})^*\ P'}{P \overset{\varepsilon}{\Longrightarrow} P'} \qquad\qquad \frac{P \overset{a_1}{\Longrightarrow} P' \quad P' \overset{a_2 \ldots a_n}{\Longrightarrow} P''}{P \overset{a_1 a_2 \ldots a_n}{\Longrightarrow} P''}$$

5.1 Defining the extensions in Maude

We show in this section how these extensions can be defined in Maude. First, we define the reflexive, transitive closure of the basic transition relation we have already implemented. We need an operator to build the new kind of judgements

```
op _‘(--_->‘)*_ : Process? Act Process? -> Judgement [prec 50] .
```

where we allow metavariables as processes because we need them for the implementation of the weak transition relation. The rewrite rules defining the closure are as follows:

```
rl [refl] :   P (-- A ->)* P
         => --------------------
                 emptyJS .

rl [refl] :  P (-- A ->)* ?P
         => -----------------
                [?P := P] .

rl [tran] :             P (-- A ->)* P?
         => -------------------------------------------
              P -- A -> ?(NEW1)P   ?(NEW1)P (-- A ->)* P? .
```

In the same way, we can define in Maude the weak transition relation both for actions and for traces:

```
op _==_==>_ : Process? Act Process? -> Judgement [prec 50] .

rl [act] :                  P == A ==> P?
        => ----------------------------------------------------
             P (-- tau ->)* ?(NEW1)P   ?(NEW1)P -- A -> ?(NEW2)P
                        ?(NEW2)P (-- tau ->)* P? .
```

```
  op _=_==>_ : Process? Trace Process? -> Judgement [prec 50] .

  rl [nil] :    P = nil ==> P?
          => --------------------
              P (-- tau ->)* P? .

  rl [seq] :                 P = A Tr ==> P?
          => --------------------------------------------
              P == A ==> ?(NEW1)P   ?(NEW1)P = Tr ==> P? .
```

And in the same way as before, we can define a function to calculate the successors of a process with respect to the weak transition semantics:

$$wsucc(P, As) = \{P' \mid P \overset{a}{\Longrightarrow} P' \wedge a \in As\}.$$

```
  op wsucc : Term TermSet -> TermSet .
  eq wsucc(T, {}) = {} .
  eq wsucc(T, A U AS) = filter(allSol(
              '`{`{_|_`}`} [ '_==_==>_ [ T, A, '?`(_`)P [ { ''proc } 'Qid ]] ,
                { 'emptyJS } 'JudgementSet ]), '?`(_`)P [ { ''proc } 'Qid ])
      U wsucc(T,AS) .
```

5.2 Examples

By continuing with our running example of module CCS-OK in Section 2.4, we can now prove that process p2 can perform an observable 'b (and we do not care about which process it becomes):

```
Maude> (red rewDepth({{ p2 == 'b ==> ?('any)P }}) .)
result Term : {{ emptyJS }}
```

We can prove that the recursive process 'Proc can execute successively three observable 'a's and become the same process:

```
Maude> (red rewDepth({{ 'Proc = 'a 'a 'a ==> 'Proc }}) .)
result Term : {{ emptyJS }}
```

Finally, we can calculate the successors of the recursively defined process 'Proc after performing action 'a allowing τ transitions:

```
Maude> (red wsucc('Proc,'a) .)
result TermSet : tau . 'Proc U 'Proc
```

6 Modal logic for CCS processes in Maude

In this section we show how we can define in Maude the semantics of a modal logic for CCS processes by using the operations of previous sections.

6.1 Hennessy-Milner logic

We introduce a *modal* logic for describing local capabilities of CCS processes that we will implement in Maude in the next section. This logic, that generalizes Hennessy-Milner logic [12], and its semantics are presented in [23]. Formulas are as follows

$$\Phi ::= \texttt{tt} \mid \texttt{ff} \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid [K]\Phi \mid \langle K\rangle\Phi$$

and the satisfaction relation describing when a process P satisfies a property Φ, $P \models \Phi$, is defined as follows

$$\begin{array}{ll}
P \models \texttt{tt} & \\
P \not\models \texttt{ff} & \\
P \models \Phi_1 \wedge \Phi_2 & \text{iff } P \models \Phi_1 \text{ and } P \models \Phi_2 \\
P \models \Phi_1 \vee \Phi_2 & \text{iff } P \models \Phi_1 \text{ or } P \models \Phi_2 \\
P \models [K]\Phi & \text{iff } \forall\, Q \in \{P' \mid P \xrightarrow{a} P' \wedge a \in K\} \,.\, Q \models \Phi \\
P \models \langle K\rangle\Phi & \text{iff } \exists\, Q \in \{P' \mid P \xrightarrow{a} P' \wedge a \in K\} \,.\, Q \models \Phi
\end{array}$$

6.2 Implementation in Maude

First, we define a sort `HMFormula` of modal logic formulas and operations to build these formulas:

```
(mod MODAL-LOGIC is
  protecting CCS-SUCC .

  sort HMFormula .

  ops tt ff : -> HMFormula .
  op _/\_ : HMFormula HMFormula -> HMFormula .
  op _\/_ : HMFormula HMFormula -> HMFormula .
  op `[_`]_ : TermSet HMFormula -> HMFormula .
  op <_>_ : TermSet HMFormula -> HMFormula .
```

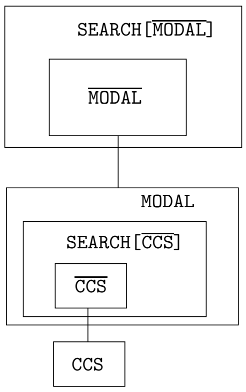
We define the modal logic semantics in the same way as we did with the CCS semantics, that is, by defining rewrite rules that rewrite a judgement $P \models \Phi$ into the set of judgements which have to be fulfilled:

```
  op _|=_ : Term HMFormula -> Judgement .

  op forall : TermSet HMFormula -> JudgementSet .
  op exists : TermSet HMFormula -> JudgementSet .

  var P : Term .
  vars K PS : TermSet .
  vars Phi Psi : HMFormula .

  rl [true] : P |= tt => emptyJS .

  rl [and] : P |= Phi /\ Psi => (P |= Phi) (P |= Psi) .
```

```
rl [or] : P |= Phi \/ Psi => P |= Phi .

rl [or] : P |= Phi \/ Psi => P |= Psi .

rl [box] : P |= [ K ] Phi => forall(succ(P, K), Phi) .

rl [diam] : P |= < K > Phi => exists(succ(P, K), Phi) .

eq forall({}, Phi) = emptyJS .
eq forall(P U PS, Phi) = (P |= Phi) forall(PS, Phi) .

rl [ex] : exists(P U PS, Phi) => P |= Phi .
```

These rules are also nondeterministic. For example, the application of the two rules `or` is nondeterministic because they have the same lefthand side, and the rule `ex` is also nondeterministic because of multiple matchings modulo associativity and commutativity.

We can instantiate the module `SEARCH` in Section 2.3 with the metarepresentation of the module containing the definition of the modal logic semantics. So we obtain the following structure of modules:



6.3 Examples

As an example taken from [23], we show some modal formulas satisfied by a vending machine `'Ven` defined in a CCS context as

```
eq context = 'Ven =def '2p . 'VenB  + '1p . 'VenL  &
             'VenB =def 'big . 'collectB . 'Ven    &
             'VenL =def 'little . 'collectL . 'Ven .
```

and how they can be proved in Maude.

One of the properties that the vending machine fulfills is that a button cannot be depressed initially, that is, before money is deposited. We can prove in Maude that `'Ven |= ['big, 'little]ff`

```
Maude> (red rewDepth('Ven |= [ 'big U 'little ] ff) .)
result Term : emptyJS
```

Another interesting property of `'Ven` is that after a `'2p` coin is inserted, the little button cannot be depressed whereas the big one can:

```
Maude> (red rewDepth('Ven |= [ '2p ](([ 'little ] ff) /\ (< 'big > tt))) .)
result Term : emptyJS
```

`'Ven` also satisfies that when a coin has been deposited no other coin can be inserted,

```
Maude> (red rewDepth('Ven |= [ '1p U '2p ] [ '1p U '2p ] ff) .)
result Term : emptyJS
```

Finally, after a coin is deposited and a button is depressed, an item (big or little) can be collected,

```
Maude> (red rewDepth(
   'Ven |= ['1p U '2p]['big U 'little]<'collectB U 'collectL>tt) .)
result Term : emptyJS
```

6.4 More modalities

If we want to give to the (silent) τ action a special status, we can introduce new modalities $[\![K]\!]$ and $\langle\!\langle K\rangle\!\rangle$ defined by means of the weak transition relation

$$\begin{array}{ll} P \models [\![K]\!]\Phi & \text{iff } \forall\, Q \in \{P' \mid P \stackrel{a}{\Longrightarrow} P' \wedge a \in K\} \,.\, Q \models \Phi \\ P \models \langle\!\langle K\rangle\!\rangle\Phi & \text{iff } \exists\, Q \in \{P' \mid P \stackrel{a}{\Longrightarrow} P' \wedge a \in K\} \,.\, Q \models \Phi \end{array}$$

The implementation in Maude of these new modalities is as follows:

```
  op `[`[_`]`]_ : TermSet HMFormula -> HMFormula .
  op <<_>>_ : TermSet HMFormula -> HMFormula .

  rl [box] : P |= [[ K ]] Phi => forall(wsucc(P, K), Phi) .
  rl [diam] : P |= << K >> Phi => exists(wsucc(P, K), Phi) .
```

where we use the operation `wsucc` to calculate the successors of process `P` with respect to the weak transition semantics.

We can now prove some properties of a railroad crossing system specified in a CCS context as follows:

```
  eq context =  'Road =def 'car . 'up . ~ 'ccross . ~ 'down . 'Road      &
                'Rail =def 'train . 'green . ~ 'tcross . ~ 'red . 'Rail  &
                'Signal =def  ~ 'green . 'red . 'Signal
                             + ~ 'up . 'down . 'Signal                   &
                'Crossing =def ('Road | ('Rail | 'Signal))
                                  \ 'green \ 'red \ 'up \ 'down    .
```

The process `'Crossing` satisfies that when a car and a train arrive to the crossing, only one of them has the possibility to cross it.

```
Maude> (red RewDepth(
   'Crossing |= [['car]][['train]]((<<~ 'ccross>> tt) \/ (<<~ 'tcross>> tt))) .)
result Term : emptyJS
Maude> (red RewDepth(
   'Crossing |= [['car]][['train]]((<<~ 'ccross>> tt) /\ (<<~ 'tcross>> tt))) .)
result Term : error*
```

7 Conclusion

We have represented the CCS structural operational semantics in rewriting logic in a general way, solving the problems of new variables in the righthand side of the rules and nondeterminism by means of the reflective features of rewriting logic and its realization in the Maude module `META-LEVEL`. In particular, we have used metavariables (together with the substitution cabability of the metalevel operation `meta-apply`) to solve the problem of new variables, and a search strategy which deals with conceptual trees of rewritings to solve the problem of nondeterminism.

We have seen how the semantics can be extended to answer questions about the capability of a process to perform an action, by considering metavariables as processes in the same way as we had done with actions. Having metavariables as processes allows extending the semantics to traces, defining weak transition semantics, and answering which are the successors of a process after performing an action. In its turn, this allows a representation of the semantics of the Hennessy-Milner modal logic in a quite similar way as it is done in its mathematical definition.

Work in progress applies all these techniques to the specification language E-LOTOS [21], developed within ISO for the formal specification of open distributed concurrent systems. We are still in the process of writing all the semantic rules for E-LOTOS in Maude, but we do not anticipate any new problems.

Acknowledgements We are very grateful to José Meseguer for his very helpful comments on an earlier version of this paper.

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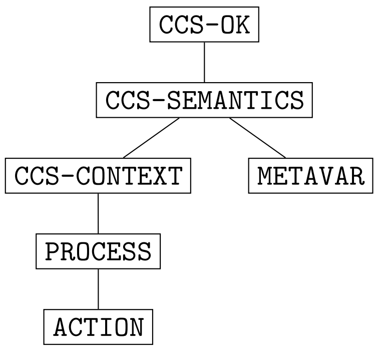
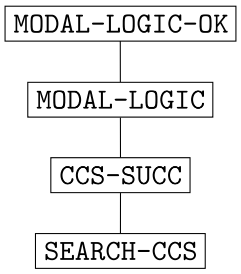
A Complete Maude specification

In order to help to understand the module structure we first present two diagrams showing module importation. The modules used to represent the CCS semantics are as follows:



The module CCS-OK (metarepresented) is used to instantiate the generic module SEARCH, and this instantiation is in module SEARCH-CCS. The modules used to represent the modal logic are as follows:



The module MODAL-LOGIC-OK (metarepresented) is used to instantiate again the generic module SEARCH, and this instantiation is in module SEARCH-MODAL-CCS.

The complete Maude specification is the following one:

```
--- an action is the silent action or a label
(fmod ACTION is
  protecting QID .
  sorts Label Act .
  subsorts Qid < Label < Act .

  op tau : -> Act .  *** silent action

  op ~_ : Label -> Label .
  var N : Label .
  eq ~ ~ N = N .
endfm)

--- CCS syntax
(fmod PROCESS is
  protecting ACTION .
  sorts ProcessId Process .
  subsorts Qid < ProcessId < Process .

  op 0 : -> Process .                              *** inaction
  op _._ : Act Process -> Process [prec 25] .      *** prefix
  op _+_ : Process Process -> Process [prec 35] .  *** summation
  op _|_ : Process Process -> Process [prec 30] .  *** composition
  op _‘[_/_‘] : Process Label Label -> Process [prec 20] .
                     *** relabelling: [b/a] relabels "a" to "b"
  op _\_ : Process Label -> Process [prec 20] .    *** restriction
endfm)

--- defining equations and contexts
(fmod CCS-CONTEXT is
  protecting PROCESS .
  sorts BadProcess Context BadContext .
  subsort Process < BadProcess .
  subsort Context < BadContext .

  op _=def_ : ProcessId Process -> Context [prec 40] .
  op nil : -> Context .
  op _&_ : BadContext BadContext -> BadContext
              [assoc comm id: nil prec 42] .
  op _definedIn_ : ProcessId Context -> Bool .
  op def : ProcessId Context -> BadProcess .
  op not-defined : -> BadProcess .

  op context : -> Context .

  vars X X’ : ProcessId .
  var P : Process .
  var C : Context .

  cmb (X =def P) & C : Context if not(X definedIn C) .

  eq X definedIn nil = false .
  eq X definedIn ((X’ =def P) & C) = (X == X’) or (X definedIn C) .

  eq def(X, nil) = not-defined .
  eq def(X, ((X =def P) & C)) = P .
  ceq def(X, ((X’ =def P) & C)) = def(X, C) if X =/= X’ .
endfm)

(fmod METAVAR is
  protecting QID .
  sorts MetaVarAct MetaVarProc .
  op ?`(_`)A : Qid -> MetaVarAct .
  op ?`(_`)P : Qid -> MetaVarProc .
endfm)

--- CCS operational semantics
(mod CCS-SEMANTICS is
  pr CCS-CONTEXT .
  pr METAVAR .

  sort Act? .
  subsort Act < Act? .
  subsort MetaVarAct < Act? .

  sort Process? .
  subsort Process < Process? .
  subsort MetaVarProc < Process? .

  vars L L' M : Label .
  vars A A' A'' : Act .
  vars A? A'? : Act? .
  vars P P' Q Q' : Process .
  vars P? P'? Q? Q'? : Process? .
  var X : ProcessId .
  vars ?A ?A' : MetaVarAct .
  vars ?P ?Q ?R : MetaVarProc .

  vars NEW1 NEW2 NEW3 : Qid .
  op how-many-mv : -> MachineInt .  *** number of metavariables used
  eq how-many-mv = 3 .

  op ~_ : Act? -> Act? .

  op _\_ : Process? Label -> Process? .
  op _`[_/_`] : Process? Label Label -> Process? .
  op _|_ : Process? Process? -> Process? .

  sort Judgement .
  op _--_->_ : Process? Act? Process? -> Judgement [prec 50] .
  op `[_:=_`] : MetaVarAct Act -> Judgement .
  op `[_=/=_`] : Act? Act? -> Judgement .
  op `[_:=_`] : MetaVarProc Process? -> Judgement .

  sort JudgementSet .

  op emptyJS : -> JudgementSet .
  subsort Judgement < JudgementSet .
  op __ : JudgementSet JudgementSet -> JudgementSet
          [assoc id: emptyJS prec 60] .

  *** commutativity
  eq (P? -- A? -> Q?) [?A := A] = [?A := A] (P? -- A? -> Q?) .
  eq (P? -- A? -> Q?) [?P := P] = [?P := P] (P? -- A? -> Q?) .
```

```
var J : Judgement .
vars JS JS' : JudgementSet .

sort Configuration .

op `{`{_`}`} : JudgementSet -> Configuration  .

op `{`{_|_`}`} : JudgementSet JudgementSet -> Configuration  .


*** Assignments MetaVarAct := Act

op <act_:=_>_ : MetaVarAct Act Act? -> Act? .
op <act_:=_>_ : MetaVarAct Act Judgement -> Judgement .
op <act_:=_>_ : MetaVarAct Act JudgementSet -> JudgementSet .

eq <act ?A := A > ?A = A .
ceq <act ?A := A > ?A' = ?A' if ?A =/= ?A' .
eq <act ?A := A > A' = A' .
eq <act ?A := A > (~ A?) = ~(<act ?A := A > A?) .

eq <act ?A := A > (Q? -- A? -> Q'?) =  (Q? -- <act ?A := A > A? -> Q'?) .
eq <act ?A := A > [ ?A' := A' ] = [?A' :=  A'] .

eq <act ?A := A > [ A? =/= A'? ] =
   [<act ?A := A > A? =/= <act ?A := A > A'?] .

eq <act ?A := A > [ ?P := P'? ] = [ ?P := P'? ] .

eq <act ?A := A > emptyJS = emptyJS .
ceq <act ?A := A > (J JS) = (<act ?A := A > J) (<act ?A := A > JS)
                         if JS =/= emptyJS .

*** Assignments MetaVarProc := Process

op <proc_:=_>_ : MetaVarProc Process Process? -> Process? .
op <proc_:=_>_ : MetaVarProc Process Judgement -> Judgement .
op <proc_:=_>_ : MetaVarProc Process JudgementSet -> JudgementSet .

eq <proc ?P := P > ?P = P .
ceq <proc ?P := P > ?Q = ?Q if ?P =/= ?Q .
eq <proc ?P := P > P' = P' .
eq <proc ?P := P > (Q? \ L) = (<proc ?P := P > Q?) \ L .
eq <proc ?P := P > (Q? [ M / L ]) = (<proc ?P := P > Q?) [ M / L ] .
eq <proc ?P := P > (P? | Q?) =
     (<proc ?P := P > P?) | (<proc ?P := P > Q?) .

eq <proc ?P := P > (Q? -- A? -> Q'?) =
((<proc ?P := P > Q?) -- A? -> (<proc ?P := P > Q'?)) .
eq <proc ?P := P > [ ?A := A ] = [ ?A := A ] .
eq <proc ?P := P > [ A? =/= A'? ] = [ A? =/= A'? ] .

eq <proc ?P := P > [ ?Q := Q? ] = [ ?Q := <proc ?P := P > Q? ] .

eq <proc ?P := P > emptyJS = emptyJS .
ceq <proc ?P := P > (J JS) = (<proc ?P := P > J) (<proc ?P := P > JS)
                         if JS =/= emptyJS .

rl [bind] : {{ [?A := A] JS }} => {{ <act ?A := A > JS }} .

rl [bind] : {{ [?A := A] JS | JS' }} =>
            {{ (<act ?A := A > JS) | [?A := A] JS' }} .

crl [dist] : [A =/= A'] => emptyJS  if A =/= A' .

rl [bind] : {{ [?P := P] JS }} => {{ <proc ?P := P > JS }} .

rl [bind] : {{ [?P := P] JS | JS' }} =>
            {{ (<proc ?P := P > JS) | [?P := P] JS' }} .


*** Prefix

rl [pref] :   A . P -- A -> P
        => ---------------------
             emptyJS .

rl [pref] :  A . P -- ?A -> P
        => --------------------
                [?A := A] .

rl [pref] :  A . P -- A -> ?P
        => --------------------
                [?P := P] .

rl [pref] :   A . P -- ?A -> ?P
        => -----------------------
             [?A := A] [?P := P] .

*** Summation

rl [sum] :   P + Q -- A -> P'
        => --------------------
               P -- A -> P' .

rl [sum] :   P + Q -- ?A -> P'
        => --------------------
               P -- ?A -> P' .

rl [sum] :   P + Q -- A -> ?P
        => --------------------
               P -- A -> ?P .
```

```
rl [sum] :   P + Q -- ?A -> ?P
        => --------------------
              P -- ?A -> ?P .

rl [sum] :   P + Q -- A -> Q'
        => --------------------
              Q -- A -> Q' .

rl [sum] :   P + Q -- ?A -> Q'
        => --------------------
             Q -- ?A -> Q' .

rl [sum] :   P + Q -- A -> ?Q
        => --------------------
              Q -- A -> ?Q .

rl [sum] :   P + Q -- ?A -> ?Q
        => --------------------
              Q -- ?A -> ?Q .


*** Composition

rl [par] :   P | Q -- A -> P' | Q
        => ------------------------
               P -- A -> P' .

rl [par] :   P | Q -- ?A -> P' | Q
        => ------------------------
               P -- ?A -> P' .

rl [par] :   P | Q -- A -> P | Q'
        => ------------------------
               Q -- A -> Q' .

rl [par] :   P | Q -- ?A -> P | Q'
        => ------------------------
               Q -- ?A -> Q' .

rl [par] :            P | Q -- tau -> P' | Q'
        => -------------------------------------------------
             P -- ?(NEW1)A -> P'   Q -- ~ ?(NEW1)A -> Q' .

rl [par] :                 P | Q -- ?A -> P' | Q'
        => ------------------------------------------------------------
            P -- ?(NEW1)A -> P'   Q -- ~ ?(NEW1)A -> Q'  [?A := tau] .

rl [par] :              P | Q -- A -> ?P
        => -------------------------------------------
             P -- A -> ?(NEW1)P  [?P := ?(NEW1)P | Q] .
```

```
rl [par] :              P | Q -- ?A -> ?P
        => --------------------------------------------
            P -- ?A -> ?(NEW1)P  [?P := ?(NEW1)P | Q] .

rl [par] :            P | Q -- A -> ?Q
        => -------------------------------------------
            Q -- A -> ?(NEW1)P  [?Q := P | ?(NEW1)P] .

rl [par] :             P | Q -- ?A -> ?Q
        => ---------------------------------------------
            Q -- ?A -> ?(NEW1)P  [?Q := P | ?(NEW1)P] .

rl [par] :                 P | Q -- tau -> ?R
        => -------------------------------------------------------
            P -- ?(NEW1)A -> ?(NEW2)P   Q -- ~ ?(NEW1)A -> ?(NEW3)P
                      [?R := ?(NEW2)P | ?(NEW3)P] .

rl [par] :                  P | Q -- ?A -> ?R
        => -------------------------------------------------------
            P -- ?(NEW1)A -> ?(NEW2)P   Q -- ~ ?(NEW1)A -> ?(NEW3)P
                  [?A := tau] [?R := ?(NEW2)P | ?(NEW3)P] .

*** Restriction

crl [res] :  P \ L -- A -> P' \ L
        => ------------------------
                 P -- A -> P'
        if (A =/= L) and (A =/= ~ L) .

rl [res] :         P \ L -- ?A -> P' \ L
       => ------------------------------------------
           P -- ?A -> P'  [?A =/= L]  [?A =/= ~ L] .

crl [res] :           P \ L -- A -> ?P
       => -------------------------------------------
           P -- A -> ?(NEW1)P [?P := ?(NEW1)P \ L]
       if (A =/= L) and (A =/= ~ L) .

rl [res] :           P \ L -- ?A -> ?P
       => --------------------------------------------
           P -- ?A -> ?(NEW1)P  [?A =/= L] [?A =/= ~ L]
                    [?P := ?(NEW1)P \ L] .

*** Relabelling

rl [rel] :  P[M / L] -- M -> P'[M / L]
        => -----------------------------
                    P -- L -> P' .

rl [rel] :  P[M / L] -- ~ M -> P'[M / L]
        => -------------------------------
                   P -- ~ L -> P' .
```

```
crl [rel] :  P[M / L] -- A -> P'[M / L]
         => -----------------------------
                   P -- A -> P'
         if (A =/= L) and (A =/= ~ L) .

rl [rel] :  P[M / L] -- ?A -> P'[M / L]
        => -----------------------------
              (P -- L -> P') [?A := M] .

rl [rel] :  P[M / L] -- ?A -> P'[M / L]
        => -------------------------------
            P -- ~ L -> P'  [?A := ~ M] .

rl [rel] :      P[M / L] -- ?A -> P'[M / L]
        => ------------------------------------------
             P -- ?A -> P' [?A =/= L] [?A =/= ~ L] .

rl [rel] :             P[M / L] -- M -> ?P
        => --------------------------------------------------
            P -- L -> ?(NEW1)P [?P := ?(NEW1)P [ M / L ]] .

rl [rel] :              P[M / L] -- ~ M -> ?P
        => ------------------------------------------------------
            P -- ~ L -> ?(NEW1)P [?P := ?(NEW1)P [ M / L ]]  .

rl [rel] :              P[M / L] -- ?A -> ?P
        => ---------------------------------------------------------
            P -- L -> ?(NEW1)P [?A := M] [?P := ?(NEW1)P [ M / L ]] .

rl [rel] :                  P[M / L] -- ?A -> ?P
        => ---------------------------------------------------------------
            P -- ~ L -> ?(NEW1)P [?A := ~ M] [?P := ?(NEW1)P [ M / L ]] .

crl [rel] :                P[M / L] -- A -> ?P
         => ------------------------------------------------
             P -- A -> ?(NEW1)P [?P := ?(NEW1)P [ M / L ]]
         if (A =/= L) and (A =/= ~ L) .

rl [rel] :             P[M / L] -- ?A -> ?P
        => --------------------------------------------
            P -- ?A -> ?(NEW1)P [?A =/= L] [?A =/= ~ L]
                    [?P := ?(NEW1)P [ M / L ]] .

*** Definition

crl [def] :         X -- A -> P'
         => ------------------------------
             def(X, context) -- A -> P'
         if (X definedIn context) .

crl [def] :         X -- ?A -> P'
```

```
          => ------------------------------
                 def(X, context) -- ?A -> P'
          if (X definedIn context) .

  crl [def] :          X -- A -> ?P
          => ------------------------------
                 def(X, context) -- A -> ?P
          if (X definedIn context) .

  crl [def] :          X -- ?A -> ?P
          => ------------------------------
                 def(X, context) -- ?A -> ?P
          if (X definedIn context) .


*** Extension to traces of actions

  sort Trace .
  subsort Act < Trace .
  op nil : -> Trace .
  op __ : Trace Trace -> Trace [assoc id: nil] .

  op _-_->_ : Process? Trace Process? -> Judgement [prec 50] .

  var Tr : Trace .

  eq (P? - Tr -> Q?) [?A := A] = [?A := A] (P? - Tr -> Q?) .
  eq (P? - Tr -> Q?) [?P := P] = [?P := P] (P? - Tr -> Q?) .


  rl [nil] :      P - nil -> P
         => ------------------
                emptyJS .

  rl [nil] :    P - nil -> ?P
         => -----------------
                   [?P := P] .

  rl [seq] :              P - A Tr -> P'
         => -----------------------------------------
                P -- A -> ?(NEW1)P   ?(NEW1)P - Tr -> P' .

  rl [seq] :              P - A Tr -> ?P
         => -----------------------------------------
                P -- A -> ?(NEW1)P   ?(NEW1)P - Tr -> ?P .

  eq <act ?A := A > (Q? - Tr -> Q'?) = (Q? - Tr -> Q'?)   .
  eq <proc ?P := P > (Q? - Tr -> Q'?) =
     ((<proc ?P := P > Q?) - Tr -> (<proc ?P := P > Q'?))   .


*** Extension to reflexive, transitive closure    (-- A ->)*
```

```
  op _‘(--_->‘)*_ : Process? Act Process? -> Judgement [prec 50] .

  eq (P? (-- A ->)* Q?) [?A := A’] = [?A := A’] (P? (-- A ->)* Q?) .
  eq (P? (-- A ->)* Q?) [?P := P] = [?P := P] (P? (-- A ->)* Q?) .

  rl [refl] :   P (-- A ->)* P
           => --------------------
               emptyJS .

  rl [refl] :  P (-- A ->)* ?P
           => -----------------
                  [?P := P] .

  rl [tran] :             P (-- A ->)* P’
           => ---------------------------------------------
               P -- A -> ?(NEW1)P   ?(NEW1)P (-- A ->)* P’ .

  rl [tran] :             P (-- A ->)* ?P
           => ---------------------------------------------
               P -- A -> ?(NEW1)P   ?(NEW1)P (-- A ->)* ?P .

  eq <act ?A := A > (P? (-- A’ ->)* P’?) = (P? (-- A’ ->)* P’?) .
  eq <proc ?P := P > (Q? (-- A ->)* Q’?) =
     ((<proc ?P := P > Q?) (-- A ->)* (<proc ?P := P > Q’?)) .


*** Extension to double arrow   == A =>

  op _==_==>_ : Process? Act Process? -> Judgement [prec 50 ] .

  eq (P? == A? ==> Q?) [?A := A] = [?A := A] (P? == A? ==> Q?) .
  eq (P? == A? ==> Q?) [?P := P] = [?P := P] (P? == A? ==> Q?) .

  rl [act] :                P == A ==> P’
          => ---------------------------------------------------
              P (-- tau ->)* ?(NEW1)P   ?(NEW1)P -- A -> ?(NEW2)P
                         ?(NEW2)P (-- tau ->)* P’ .

  rl [act] :                P == A ==> ?P
          => ---------------------------------------------------
              P (-- tau ->)* ?(NEW1)P   ?(NEW1)P -- A -> ?(NEW2)P
                         ?(NEW2)P (-- tau ->)* ?P .

  eq <act ?A := A > (P? == A’ ==> P’?) = (P? == A’ ==> P’?) .
  eq <proc ?P := P > (Q? == A ==> Q’?) =
     ((<proc ?P := P > Q?) == A ==> (<proc ?P := P > Q’?)) .


  *** Extension to double arrow with traces  = Tr ==>

  op _=_==>_ : Process? Trace Process? -> Judgement [prec 50] .
  eq (P? = A? ==> Q?) [?A := A] = [?A := A] (P? = A? ==> Q?) .
  eq (P? = A? ==> Q?) [?P := P] = [?P := P] (P? = A? ==> Q?) .

  rl [nil] :    P = nil ==> P'
          => --------------------
               P (-- tau ->)* P' .

  rl [nil] :    P = nil ==> ?P
          => ---------------------
               P (-- tau ->)* ?P .

  rl [seq] :             P = A Tr ==> P'
          => -------------------------------------------
               P == A ==> ?(NEW1)P   ?(NEW1)P = Tr ==> P' .

  rl [seq] :             P = A Tr ==> ?P
          => -------------------------------------------
               P == A ==> ?(NEW1)P   ?(NEW1)P = Tr ==> ?P .

  eq <act ?A := A > (P? = Tr ==> P'?) = (P? = Tr ==> P'?) .
  eq <proc ?P := P > (Q? = Tr ==> Q'?) =
     ((<proc ?P := P > Q?) = Tr ==> (<proc ?P := P > Q'?)) .

endm)

(fth AMODULE is
  including META-LEVEL .
  op MOD : -> Module .
  op num-metavar : -> MachineInt .
  op labels : -> QidList .
endfth)

(fmod SEARCH[M :: AMODULE] is
  sort TermSet .
  subsort Term < TermSet .

  sorts Pair PairSeq .
  subsort Pair < PairSeq .

  vars F C S OP OP' : Qid .
  vars T T' : Term .
  vars TL Before After : TermList .
  vars M N : MachineInt .
  var L : Qid .
  var LS : QidList .
  var SB : Substitution .
  var TS : TermSet .
  var PS : PairSeq .

  *** Extension of TermList with an identity element
  op ~ : -> TermList .
```

```
eq ~, TL = TL .
eq TL, ~ = TL .

op extTerm : ResultPair -> Term .
op extSubst : ResultPair -> Substitution .

eq extTerm({T, SB}) = T .
eq extSubst({T, SB}) = SB .

*** creation of new metavariables

op new-var : MachineInt -> Term .
eq new-var(N) = {conc('' , index(' , N))}'Qid .

op createSubst : MachineInt MachineInt -> Substitution .

eq createSubst(0, M) = none .
ceq createSubst(N, M) = (createSubst(_-_(N,1), M) ;
                        (conc('NEW , conc(index(' , N), '@Qid))
                         <- new-var(M + N))) if N =/= 0 .


op meta-apply' : Term Qid MachineInt MachineInt -> Term .
op allRew : Term QidList MachineInt  -> TermList .
op topRew : Term Qid MachineInt MachineInt -> TermList .
op lowerRew : Term Qid MachineInt -> TermList .
op rewArguments : Qid TermList TermList Qid MachineInt -> TermList .
op rebuild : Qid TermList TermList TermList -> TermList .

eq meta-apply'(T, L, N, M) =
    extTerm(meta-apply(MOD, T, L, createSubst(num-metavar, M), N)) .

eq allRew(T, nil, M) = ~ .
eq allRew(T, L LS, M) = topRew(T, L, 0, M) ,
                         lowerRew(T, L, M) , allRew(T, LS, M) .

eq topRew(T, L, N, M) =
    if meta-apply'(T, L, N, M) == error* then ~
    else (meta-apply'(T, L, N, M) , topRew(T, L, N + 1, M))
    fi .

eq lowerRew({C}S, L, M) = ~ .
eq lowerRew(OP[TL], L, M) = rewArguments(OP, ~, TL, L, M) .

eq rewArguments(OP, Before, T, L, M) =
    rebuild(OP, Before, allRew(T, L, M), ~) .
eq rewArguments(OP, Before, (T, After), L, M) =
    rebuild(OP, Before, allRew(T, L, M), After) .


eq rebuild(OP, Before, ~, After) = ~ .
eq rebuild(OP, Before, T, After) = meta-reduce(MOD, OP[Before, T, After]) .
eq rebuild(OP, Before, (T, TL), After) =
    meta-reduce(MOD, OP[Before, T, After]) ,
    rebuild(OP, Before, TL, After) .


*** Term sets

op `{`} : -> TermSet .
op _U_ : TermSet TermSet -> TermSet [assoc comm id: {}] .
op _isIn_ : Term TermSet -> Bool .

ceq T U T' = T if meta-reduce(MOD, '_==_[T, T']) == {'true}'Bool .
eq T isIn {} = false .
ceq T isIn (T' U TS) = true
     if meta-reduce(MOD, '_==_[T, T']) == {'true}'Bool .
ceq T isIn (T' U TS) = T isIn TS
     if meta-reduce(MOD, '_=/=_[T, T']) == {'true}'Bool .


*** Definition of the depth-first strategy
*** (with substitution of three "metavariables")

op <_`,_> : Term MachineInt -> Pair .
op nil : -> PairSeq .
op _|_ : PairSeq PairSeq -> PairSeq [assoc id: nil] .

op buildPairs : TermList MachineInt -> PairSeq .

eq buildPairs(~, N) = nil .
eq buildPairs(T, N) = < T , N > .
eq buildPairs((T, TL), N) = < T , N > | buildPairs(TL, N) .


op rewDepth : Term -> Term .
op rewDepth' : PairSeq -> Term .

eq rewDepth(T) = rewDepth'(< meta-reduce(MOD, T), 0 >) .

eq rewDepth'(nil) = error* .
eq rewDepth'(< T , N > | PS) =
    if meta-reduce(MOD, 'ok[T]) == {'solution}'Answer then T
    else (if meta-reduce(MOD, 'ok[T]) == {'no-solution}'Answer then
                  rewDepth'(PS)
          else rewDepth'(buildPairs(allRew(T, labels, N),
                                    (N + num-metavar)) | PS )
          fi)
    fi .

*** All solutions

op allSol : Term -> TermSet .
```

```
  eq allSol(T) = allSolDepth(< meta-reduce(MOD,T) , 0 >) .

  op allSolDepth : PairSeq -> TermSet .

  eq allSolDepth(nil) = {} .
  eq allSolDepth(< T , N > | PS) =
    if meta-reduce(MOD, 'ok[T]) == {'solution}'Answer then
                (T U allSolDepth(PS))
    else (if meta-reduce(MOD, 'ok[T]) == {'no-solution}'Answer then
                   allSolDepth(PS)
         else allSolDepth(buildPairs(allRew(T, labels, N),
                                                (N + num-metavar)) |
                                      PS)
         fi)
    fi .

endfm)

(mod CCS-OK is
  protecting CCS-SEMANTICS .

  ops p1 p2 p3 : -> Process .

  eq p1 = ('a . 0) + ('b . 0 | ('c . 0 + 'd . 0)) .
  eq p2 = ('a  . 'b . 0 | (~ 'c . 0) ['a / 'c]) \ 'a .
  eq p3 = ('Proc | ~ 'a . 'b . 0) \ 'a .

  sort Answer .
  ops solution no-solution maybe-sol : -> Answer .

  op ok : Configuration -> Answer .

  vars A A' : Act .
  var P : Process .
  var P? : Process .
  vars JS JS' : JudgementSet .

  eq ok({{ emptyJS }}) = solution .
  ceq ok({{ JS }}) = maybe-sol if JS =/= emptyJS .

  eq ok({{ emptyJS | JS' }}) = solution .
  ceq ok({{ JS | JS'}}) = maybe-sol if JS =/= emptyJS .

  eq context = (('Proc   =def  'a . tau . 'Proc) &
               ('Ven    =def  '2p . 'VenB  +  '1p . 'VenL)  &
               ('VenB   =def  'big . 'collectB . 'Ven)    &
               ('VenL   =def  'little . 'collectL . 'Ven)   &
               ('Road   =def  'car . 'up . ~ 'ccross . ~ 'down . 'Road)  &
               ('Rail   =def  'train . 'green . ~ 'tcross . ~ 'red . 'Rail) &
               ('Signal =def  ~ 'green . 'red . 'Signal
                            + ~ 'up . 'down . 'Signal)  &
               ('Crossing =def (('Road | ('Rail | 'Signal))
                                  \ 'green \ 'red \ 'up \ 'down )))  .
endm)

(mod META-CCS is
  including META-LEVEL .
  op METACCS : -> Module .
  eq METACCS = up(CCS-OK) .
  op labelsIDs : -> QidList .
  eq labelsIDs = ( 'bind 'dist 'pref 'sum 'par 'res 'rel 'def 'nil 'seq
                   'refl 'tran 'act ) .
endm)

(view ModuleCCS from AMODULE to META-CCS is
  op MOD to METACCS .
  op num-metavar to 3 .
  op labels to labelsIDs .
endv)

(mod SEARCH-CCS is
  protecting SEARCH[ModuleCCS] .
endm)

(fmod CCS-SUCC is
  pr SEARCH-CCS .

  op succ : Term TermSet -> TermSet .
  op succTr : Term TermSet -> TermSet .

  op wsucc : Term TermSet -> TermSet .
  op wsuccTr : Term TermSet -> TermSet .

  op filter : TermSet TermSet -> TermSet .
  op filter-bind : TermList TermSet -> TermSet .

  var A T T' BS : Term .
  vars MVS TS AS : TermSet .
  var TL : TermList .

  eq filter({}, MVS) = {} .

  eq filter(_U_( '`{`{_|_`}`} [ { 'emptyJS } 'JudgementSet ,
                               BS ],
            TS), MVS) = _U_(filter-bind(BS, MVS), filter(TS, MVS)) .

  eq filter-bind( ('`[_:=_`] [T, T']) , MVS) =
     if (T isIn MVS) then T' else {} fi .

  eq filter-bind( ('`[_:=_`] [T, T'], TL) , MVS) =
     if (T isIn MVS) then _U_(T', filter-bind(TL, MVS))
     else filter-bind(TL, MVS)
     fi .
  eq filter-bind('__ [ TL ] , MVS) = filter-bind(TL, MVS) .

  eq succ(T, {}) = {} .

  eq succ(T, A U AS) = filter(allSol(
              '`{`{_|_`}`} [ '_--_->_ [ T, A, '?`(_`)P [ { ''proc } 'Qid ]] ,
                { 'emptyJS } 'JudgementSet ]),
              '?`(_`)P [ { ''proc } 'Qid ])
       U succ(T,AS) .

  eq succTr(T, {}) = {} .

  eq succTr(T, _U_(A, AS)) = _U_(
       filter(allSol(
                '`{`{_|_`}`} [ '_-_->_ [ T, A, '?`(_`)P [ { ''proc } 'Qid ]] ,
                { 'emptyJS } 'JudgementSet ]),
              '?`(_`)P [ { ''proc } 'Qid ]) ,
       succTr(T,AS)) .

  eq wsucc(T, {}) = {} .
  eq wsucc(T, A U AS) = filter(allSol(
         '`{`{_|_`}`} ['_==_==>_[T, A, '?`(_`)P[{''proc}'Qid]],
         {'emptyJS}'JudgementSet]), '?`(_`)P[{''proc}'Qid])
      U wsucc(T,AS) .

  eq wsuccTr(T, {}) = {} .

  eq wsuccTr(T, _U_(A, AS)) = _U_(
       filter(allSol(
                '`{`{_|_`}`} [ '_=_==>_ [ T, A, '?`(_`)P [ { ''proc } 'Qid ]] ,
                { 'emptyJS } 'JudgementSet ]),
              '?`(_`)P [ { ''proc } 'Qid ]) ,
       wsuccTr(T,AS)) .
endfm)

(mod MODAL-LOGIC is
  protecting CCS-SUCC .

  sort HMFormula .

  ops tt ff : -> HMFormula .
  op _/\_ : HMFormula HMFormula -> HMFormula .
  op _\/_ : HMFormula HMFormula -> HMFormula .
  op `[_`]_ : TermSet HMFormula -> HMFormula .
  op <_>_ : TermSet HMFormula -> HMFormula .
  op `[`[_`]`]_ : TermSet HMFormula -> HMFormula .
  op <<_>>_ : TermSet HMFormula -> HMFormula .

  op forall : TermSet HMFormula -> JudgementSet .
  op exists : TermSet HMFormula -> JudgementSet .

  sort Judgement .
  op _|=_ : Term HMFormula -> Judgement .

  sort JudgementSet .

  op emptyJS : -> JudgementSet .
  subsort Judgement < JudgementSet .
  op __ : JudgementSet JudgementSet -> JudgementSet
          [assoc id: emptyJS] .

  var P : Term .
  var K PS : TermSet .
  vars Phi Psi : HMFormula .

  rl [true] : P |= tt => emptyJS .

  rl [and] : P |= Phi /\ Psi => (P |= Phi) (P |= Psi) .

  rl [or] : P |= Phi \/ Psi => P |= Phi .

  rl [or] : P |= Phi \/ Psi => P |= Psi .

  rl [box] : P |= [ K ] Phi => forall(succ(P, K), Phi) .

  rl [diam] : P |= < K > Phi => exists(succ(P, K), Phi) .

  eq forall({}, Phi) = emptyJS .
  eq forall(P U PS, Phi) = (P |= Phi) forall(PS, Phi) .

  rl [ex] : exists(P U PS, Phi) => P |= Phi .

  rl [box] : P |= [[ K ]] Phi => forall(wsucc(P, K), Phi) .

  rl [diam] : P |= << K >> Phi => exists(wsucc(P, K), Phi) .

endm)

(mod MODAL-LOGIC-OK is
  protecting MODAL-LOGIC .

  sort Answer .

  ops solution no-solution maybe-sol : -> Answer .

  op ok : JudgementSet -> Answer .

  var JS : JudgementSet .

  eq ok(emptyJS) = solution .
  ceq ok(JS) = maybe-sol if JS =/= emptyJS .
endm)

(mod META-MODAL-LOGIC-OK is
  including META-LEVEL .
  op METAMODALCCS : -> Module .
  eq METAMODALCCS = up(MODAL-LOGIC-OK) .
  op labelsIDs : -> QidList .
  eq labelsIDs = ( 'true 'and 'or 'box 'diam 'ex 'box 'diam ) .
endm)

(view ModuleMODAL-LOGIC-OK from AMODULE to META-MODAL-LOGIC-OK is
  op MOD to METAMODALCCS .
  op num-metavar to 0 .
  op labels to labelsIDs .
endv)

(mod SEARCH-MODAL-CCS is
  protecting SEARCH[ModuleMODAL-LOGIC-OK] .
endm)
```

B Examples in Maude

The examples used in this paper as they have are introduced in the Maude system are as follows:

```
*** Section 2.1

(select CCS-SEMANTICS .)

(rew {{ 'a . 'b . 0 -- 'a -> 'b . 0 }} .)

(rew {{ 'a . 0 + 'b . 0 -- 'b -> 0 }} .)


*** Section 2.3

(select SEARCH-CCS .)

(red allRew(up(CCS-OK, {{ 'a . 0 + 'b . 0 -- 'b -> 0 }}), labelsIDs, 0) .)


*** Section 2.4

(red rewDepth(up(CCS-OK,{{ p1 -- 'c -> ('b . 0 | 0) }})) .)

(red rewDepth(up(CCS-OK,{{ p2 -- 'a -> ('b . 0 | (~ 'c . 0) ['a / 'c]) \ 'a }})) .)

(red rewDepth(up(CCS-OK,{{ p2 -- tau -> ('b . 0 | 0 ['a / 'c]) \ 'a }})) .)

(red rewDepth(up(CCS-OK,{{ p3 -- tau -> (tau . 'Proc | 'b . 0) \ 'a }})) .)
```

```
*** Section 3.2

(red rewDepth(up(CCS-OK, {{ p1 -- 'c -> ?('any)P }})) .)

(red rewDepth(up(CCS-OK, {{ p2 -- 'a -> ?('any)P }})) .)


*** Section 3.3

(select CCS-SUCC .)

(red succ(up(CCS-OK, p1), up(CCS-OK, 'a) U up(CCS-OK, 'b)) .)


*** Section 5.2

(red rewDepth(up(CCS-OK, {{ p2 == 'b ==> ?('any)P }})) .)

(red rewDepth(up(CCS-OK, {{ 'Proc = 'a 'a 'a ==> 'Proc }})) .)

(red wsucc(up(CCS-OK, 'Proc), up(CCS-OK, 'a)) .)


*** Section 6.3

(select SEARCH-MODAL-CCS .)

(red rewDepth(up(MODAL-LOGIC-OK,
  up(CCS-OK, 'Ven) |= [up(CCS-OK, 'big) U up(CCS-OK, 'little)] ff)) .)

(red rewDepth(up(MODAL-LOGIC-OK,
  up(CCS-OK, 'Ven) |= [up(CCS-OK, '2p)](([up(CCS-OK, 'little)] ff) /\
                                         (< up(CCS-OK, 'big) > tt)))) .)

(red rewDepth(up(MODAL-LOGIC-OK,
  up(CCS-OK, 'Ven) |= [up(CCS-OK, '1p) U up(CCS-OK, '2p)]
                      [up(CCS-OK, '1p) U up(CCS-OK, '2p)] ff)) .)

(red rewDepth(up(MODAL-LOGIC-OK,
  up(CCS-OK, 'Ven) |= [up(CCS-OK, '1p) U up(CCS-OK, '2p)]
                      [up(CCS-OK, 'big) U up(CCS-OK, 'little)]
                      < up(CCS-OK, 'collectB) U up(CCS-OK, 'collectL) > tt)) .)

(red rewDepth(up(MODAL-LOGIC-OK,
  up(CCS-OK, 'Crossing) |= [[up(CCS-OK, 'car)]]
                           [[up(CCS-OK, 'train)]]
                           ( (<< up(CCS-OK, ~ 'ccross) >> tt) \/
                             (<< up(CCS-OK, ~ 'tcross) >> tt)))) .)

(red rewDepth(up(MODAL-LOGIC-OK,
  up(CCS-OK, 'Crossing) |= [[up(CCS-OK, 'car)]]
                           [[up(CCS-OK, 'train)]]
( (<< up(CCS-OK, ~ 'ccross) >> tt) /\
  (<< up(CCS-OK, ~ 'tcross) >> tt)))) .)
```